OCR RECOGNISING ACHIEVEMENT

Monday 23 January 2012 – Morning

A2 GCE MATHEMATICS

4730 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4730
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Turn over

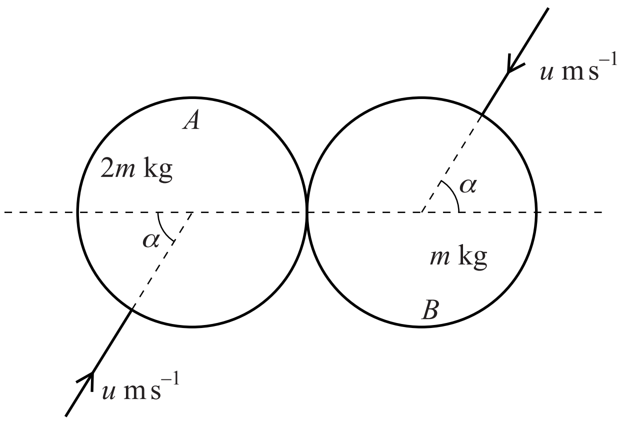
1 A particle P of mass 0.05 kg is moving on a smooth horizontal surface with speed $2\,\mathrm{m\,s^{-1}}$, when it is struck by a horizontal blow in a direction perpendicular to its direction of motion. The magnitude of the impulse of the blow is I N s. The speed of P after the blow is $2.5\,\mathrm{m\,s^{-1}}$.

(i) Find the value of I. **[4]**

Immediately before the blow P is moving parallel to a smooth vertical wall. After the blow P hits the wall and rebounds from the wall with speed $\sqrt{5}\,\mathrm{m\,s^{-1}}$.

(ii) Find the coefficient of restitution between P and the wall. **[4]**

2



Two uniform smooth spheres A and B, of equal radius, have masses $2m$ kg and m kg respectively. They are moving in opposite directions on a horizontal surface and they collide. Immediately before the collision, each sphere has speed $u\,\mathrm{m\,s^{-1}}$ in a direction making an angle α with the line of centres (see diagram). The coefficient of restitution between A and B is 0.5.

(i) Show that the speed of B is unchanged as a result of the collision. **[5]**

(ii) Find the direction of motion of each of the spheres after the collision. **[3]**

3 A particle P of mass 0.3 kg is projected horizontally with speed $u\,\mathrm{m\,s^{-1}}$ from a fixed point O on a smooth horizontal surface. At time t s after projection P is x m from O and is moving with speed $v\,\mathrm{m\,s^{-1}}$. There is a force of magnitude $1.2v^3$ N resisting the motion of P.

(i) Find an expression for $\frac{\mathrm{d}v}{\mathrm{d}x}$ in terms of v and hence show that $v = \frac{u}{4ux + 1}$. **[5]**

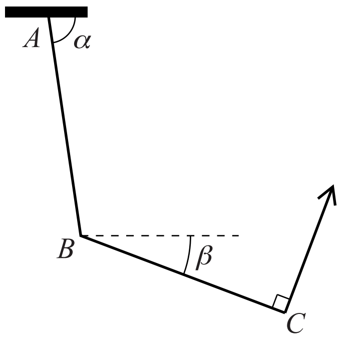
(ii) Given that $x = 2$ when $t = 9$ find the value of u. **[4]**

4 One end of a light elastic string, of natural length 0.75 m and modulus of elasticity 44.1 N, is attached to a fixed point O. A particle P of mass 1.8 kg is attached to the other end of the string. P is released from rest at O and falls vertically. Assuming there is no air resistance, find

(i) the extension of the string when P is at its lowest position, **[4]**

(ii) the acceleration of P at its lowest position. **[4]**

5



Two uniform rods AB and BC, each of length $2L$ m and of weight 84.5 N, are freely jointed at B, and AB is freely jointed to a fixed point at A. The rods are held in equilibrium in a vertical plane by a light string attached at C and perpendicular to BC. The rods AB and BC make angles α and β to the horizontal, respectively (see diagram). It is given that $\cos\beta = \frac{12}{13}$.

(i) Find the tension in the string. **[3]**

(ii) Hence show that the force acting on BC at B has horizontal component of magnitude 15 N and vertical component of magnitude 48.5 N, and state the direction of the component in each case. **[4]**

(iii) Find α. **[4]**

6 A particle P starts from rest at a point A and moves in a straight line with simple harmonic motion. At time t s after the motion starts, P's displacement from a point O on the line is x m towards A. The particle P returns to A for the first time when $t = 0.4\pi$. The maximum speed of P is $4\,\mathrm{m\,s^{-1}}$ and occurs when P passes through O.

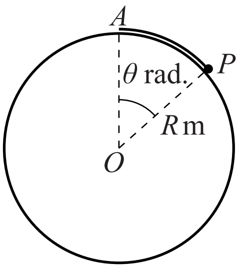
(i) Find the distance OA. **[4]**

(ii) Find the value of x and the velocity of P when $t = 1$. **[4]**

(iii) Find the number of occasions in the interval $0 < t < 1$ at which P's speed is the same as that when $t = 1$, and find the corresponding values of x and t. **[5]**

[Question 7 is printed overleaf.]

7



One end of a light elastic string, of natural length $\frac{2}{3}R$ m and with modulus of elasticity $1.2mg$ N, is attached to the highest point A of a smooth fixed sphere with centre O and radius R m. A particle P of mass m kg is attached to the other end of the string and is in contact with the surface of the sphere, where the angle AOP is equal to θ radians (see diagram).

(i) Given that P is in equilibrium at the point where $\theta = \alpha$, show that $1.8\alpha - \sin\alpha - 1.2 = 0$. Hence show that $\alpha = 1.18$ correct to 3 significant figures. **[7]**

P is now released from rest at the point of the surface of the sphere where $\theta = \frac{2}{3}$, and starts to move downwards on the surface. For an instant when $\theta = \alpha$,

(ii) state the direction of the acceleration of P, **[1]**

(iii) find the magnitude of the acceleration of P. **[7]**

Question			Answer	Marks	Guidance	
1	(i)		Triangle of velocities/momentum All correct Use of Pythagoras' theorem to find *I* $I = 0.075$	 M1 A1 M1 A1 **[4]**	For right angled triangle with at least one side correctly shown (2.5, 2, 20*I* or 0.125, 0.1, *I*) or vector equation $(v_1, v_2) = (0, 20I) + (2, 0)$ with at least 3 of the 4 components on the RHS correct $400I^2 + 2^2 = 2.5^2$ or $I^2 = 0.125^2 - 0.1^2$	 may be implied by $v_1^2 + v_2^2 = 2.5^2$ or $\sin\alpha = 0.6$
1	(ii)		Components of velocity parallel to the wall before and after are 2 and 2 Components of velocity perpendicular to the wall before and after are (–) 1.5 and 1.5*e* $[2^2 + (1.5e)^2 = 5]$ Coefficient is $\frac{2}{3}$ or 0.667	 B1 B1 M1 A1 **[4]**	 For using $v_1^2 + v_2^2 = 5$ Must be perp to wall	 may be implied
2	(i)		$2mu\cos\alpha - mu\cos\alpha = 2ma + mb$ $0.5(u\cos\alpha + u\cos\alpha) = b - a$ Comp of *B*'s velocity along l.o.c. is $u\cos\alpha$ Establishing B's speed unchanged	M1 M1 A1 A1ft A1 **[5]**	For using the p.c.m. parallel to l.o.c. For using NEL parallel to l.o.c. for both p.c.m and NEL correct & consistent dep on M1M1 gained by stating vel perp l.o.c. still $u\sin\alpha$, hence result, dep on all previous marks	allow sign errors, *m*/2*m*, sin/cos allow sign errors, e left in or by showing speed is still *u* condone 'vertical' in this part
2	(ii)		$a = 0$ correct interpretation of direction of *A* Direction of *B* is at angle α to l.o.c.., with an indication that removes ambiguity (eg in sketch)	B1 B1 B1 **[3]**	may be shown in (i) perp to l.o.c.	 condone 'vertical' for perpendicular, accept sketch, and refs to sketch in (i)

Question			Answer	Marks	Guidance	
3	(i)		 $0.3v(\mathrm{d}v/\mathrm{d}x) = -1.2v^3$ $[-v^{-1} = -4x + A]$ $[-u^{-1} = 0 + A]$ $v = \dfrac{u}{4ux+1}$	M1 A1 M1* *M1 A1 **[5]**	For using Newton's second law and $a = v(\mathrm{d}v/\mathrm{d}x)$ For finding $\mathrm{d}v/\mathrm{d}x$ in terms of v and attempting to integrate For using $v(0) = u$ AG	allow missed – sign / stray g / missed 0.3 allow $A/v = Bx + C$ oe
3	(ii)		$\int (4ux+1)dx = \int u dt$ $2ux^2 + x = ut + B$ $[(2 \times 4 - 9)u = -2]$ $u = 2$	M1* A1 *M1 A1 **[4]**	For using $v = \mathrm{d}x/\mathrm{d}t$, separating the variables and attempting to integrate one side For using $x(0) = 0$ (may be implied by absence of B) and $x(9) = 2$ – dep on int being done	$-1.2v^3=0.3\ \mathrm{d}v/\mathrm{d}t$ and attempt to int one side M1* $8t = 1/v^2 - 1/u^2$ and subst for v A1 then as main scheme
4	(i)		EE gain = $44.1x^2 \div (2\text{x}0.75)$ PE loss = $1.8g(0.75 + x)$ $[x^2 - 0.6x - 0.45 = 0]$ Extension is 1.03 m	B1 B1 M1 A1 **[4]**	 ignore signs For using EE gain = PE loss	allow use of $(e + x)$ for x $44.1x^2-26.46x-19.845=0$ allow sign errors 1.0348469...
4	(ii)		 $\dfrac{44.1\times1.03}{0.75} - 1.8 \times 9.8 = -1.8\ddot{x}$ Acceleration is $-24.0\ \mathrm{ms}^{-2}$	M1 M1 A1ft A1 **[4]**	For using $T = \lambda x/L$ For using Newton's 2^{nd} law ft their '1.03' from (i) direction must be clear	 allow missed g, m, sign error allow sign error 1.03 → − 23.84666 1.035 → − 24.01

Question			Answer	Marks	Guidance	
5	(i)		 $84.5 \times 12L/13 = T(2L)$ Tension is 39 N	M1 A1 A1 **[3]**	For taking moments about B for BC must use 12/13 for cos β	must be 2 terms involving T, L, 84.5 and sin/cos β
5	(ii)		 $X = 39 \times 5/13$ $Y = 84.5 - 39 \times 12/13$ X is to the left and Y is upwards	 M1 A1 FT A1 FT A1cao **[4]**	For resolving forces on BC horiz or vert explicit expression for X explicit expression for Y AG (numerical values – must be correct) dep M1A1A1	must involve their T and sin/cos β accept on diagram
5	(iii)		 $84.5 \times L\cos\alpha + 48.5 \times 2L\cos\alpha = 15 \times 2L\sin\alpha$ [$\tan\alpha = \frac{84.5 + 97}{30}$] $\alpha = 1.41^c$ or 80.6^o	M1* A1 *M1 A1 **[4]**	For taking moments about A for AB For obtaining a numerical expression for tan α	must involve 3 terms, 84.5, 48.5, 15,sinα and cos α; allow sign errors, L/2L similar scheme for those who take moments about A for whole system
6	(i)		[$0.4\pi = 2\pi/n$] $n = 5$ Distance OA is 0.8 m	M1 A1 M1 A1 **[4]**	For using $T = 2\pi/n$ For using $v_{max} = n(OA)$	
6	(ii)		[$x = 0.8\cos(5\times1)$] $x = 0.227$ [$\dot{x} = -0.8 \times 5\sin(5\times1)$] Velocity is 3.84 ms^{-1}	M1 A1 M1 A1 **[4]**	For using $x = a\cos nt$ For using $\dot{x} = -an\sin nt$	 Use of $v^2 = n^2(a^2 - x^2)$ M1 Direc needs to be shown for A1

Question		Answer	Marks	Guidance	
6	(iii)	t and x for one point t and x for second point t and x for third point correctly stating precisely 3 points If B1 or B0 scored (out of first 4) on above scheme, allow, subject to max mark 2, Number of occasions is 3	B2 B1 B1 B1 (M1) (A1) **[5]**	Values of t are = 0.257, 0.372, 0.885 Values of x are 0.227, –0.227 , –0.227 sc all 3 x values B2 all 3 t values B2 one t value B1 one x value B1 For $t = 1 \approx 0.8T \rightarrow 3/4T < 1 < 4/4T$ or equiv	$0.4\pi - 1$, $1 - 0.2\pi$, $0.6\pi - 1$ ignore ref to point when t = 1 can show on graph
7	(i)	Tension in string $T = mg\sin\alpha$ For using $e = R\alpha - 2R/3$ $1.8\alpha - \sin\alpha - 1.2 = 0$ Finding f(1.175) and f(1.185) correctly correct conclusion	M1 B1 B1 A1 M1 A1 A1 **[7]**	For using $T = \lambda x/L$ $mg\sin\alpha = 1.2mg\left(R\alpha - \frac{2R}{3}\right) \div \frac{2R}{3}$ AG establish result ≈ -0.008, and $\approx +0.0065$ AG $\alpha = 1.18$ correct to 3 significant figures	By iteration $\alpha = (1.2 + \sin\alpha)/1.8$ M1 start [1, 2], and 1 iteration A1 at least 1 more iteration, and conclusion 1.18(0427) A1
7	(ii)	Direction is towards O	B1 **[1]**		
7	(iii)	Gain in EE = $1.2mg(1.18R - 2R/3)^2 \div (2\text{x}2R/3)$ PE loss = $mgR(\cos 2/3 - \cos 1.18)$ $v^2 =$ $2gR[\cos 2/3 - \cos 1.18 - 0.9(1.18 - 2/3)^2]$ Acceleration is 3.29 ms^{-2}.	M1* A1 A1 M1 A1 *M1 A1 **[7]**	For using EE = $\lambda e^2 \div (2L)$ and PE = mgh ignore signs For using ½ mv^2 = PE loss – EE gain For using acceleration = v^2/R	allow α for 1.18 for A1A1 allow sign errors need 1.18 here If candidates use $mR\ddot{\theta}$ use equivalent scheme